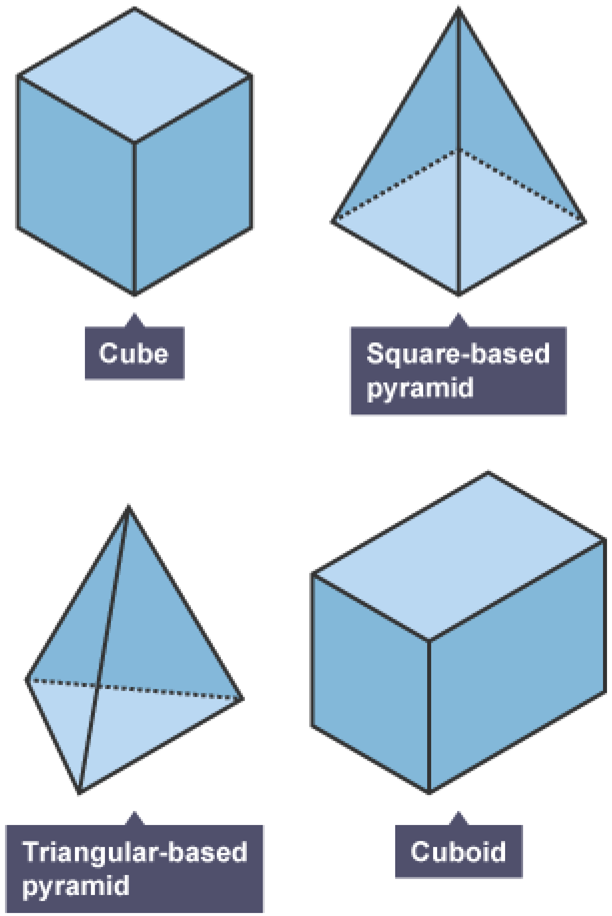
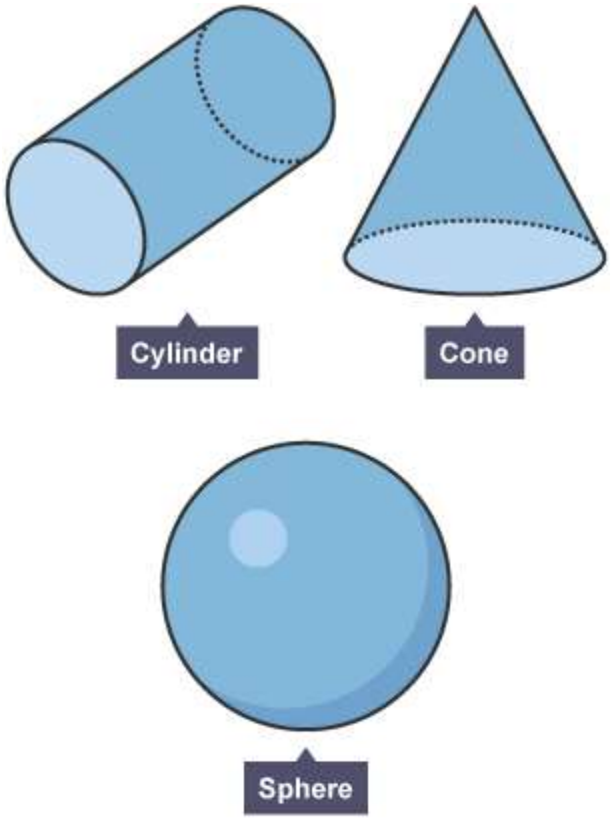
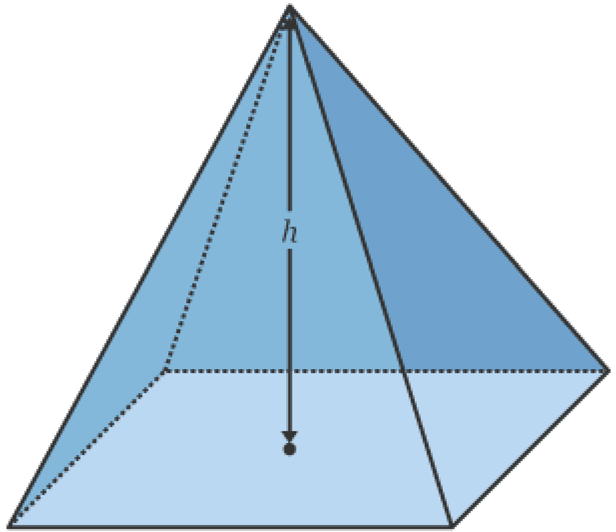
Orthogonality in space

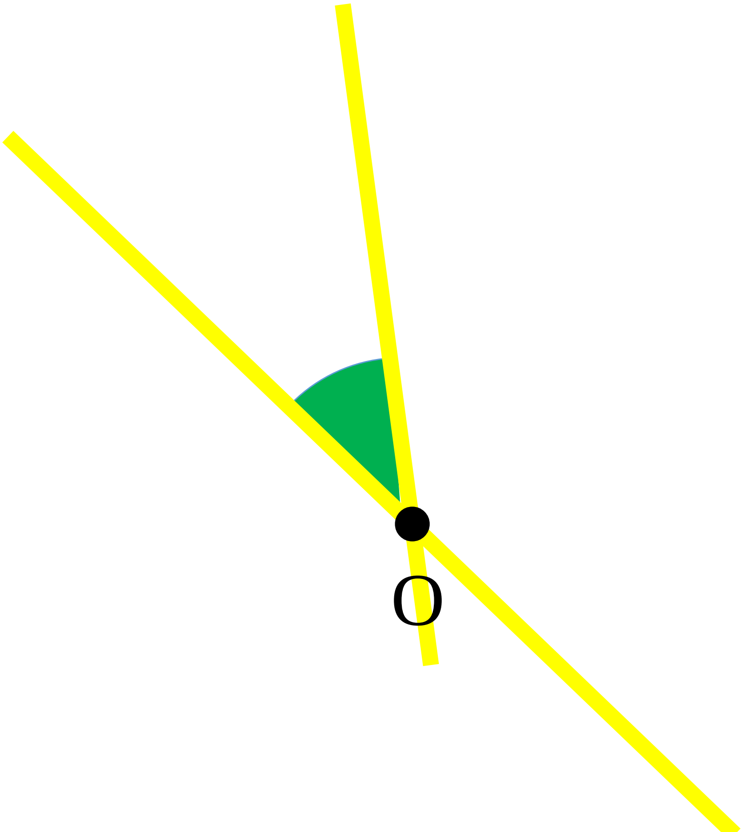






Orthogonality in space
Angle between two lines

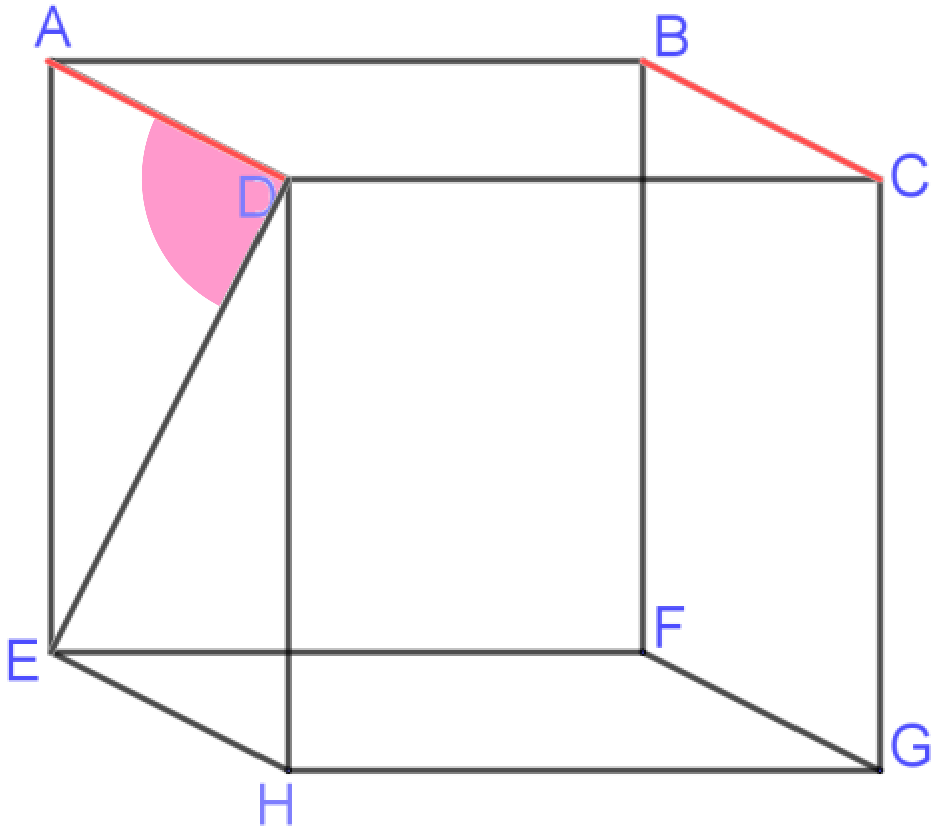
From a given point O, draw the parallel to each line.
The angle between these two parallel is equal the angle between the two main lines.



Orthogonality in space
Angle between two lines

Example:

ABCDEFGH is a cube of edge a. Determine the angle between the lines (BC) and (ED).



ABCD is a square so, (BC) // (AD).
Then:

$$(\widehat{(BC),(DE)}) = (\widehat{(AD),(DE)}) = \widehat{ADE}$$

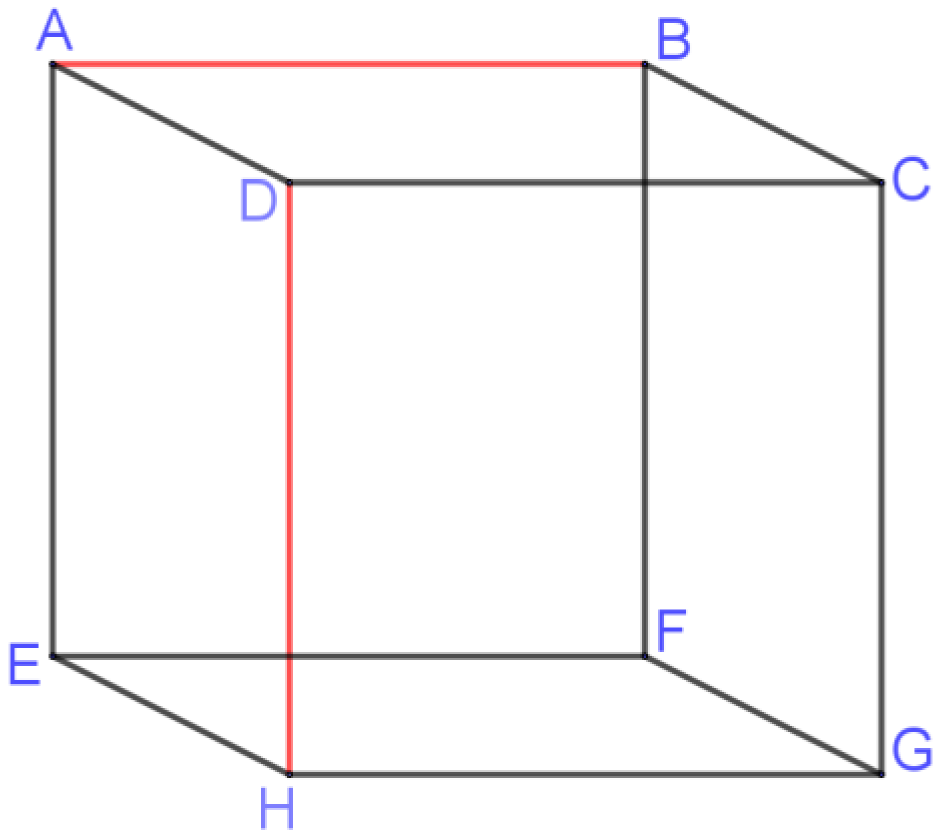
But ADHE is a square and the diagonal [DE] in the square is a bisector so, $\widehat{ADE}$

$$= \frac{\widehat{ADH}}{2} = \frac{90°}{2} = 45°$$

Orthogonality in space
Angle between two lines

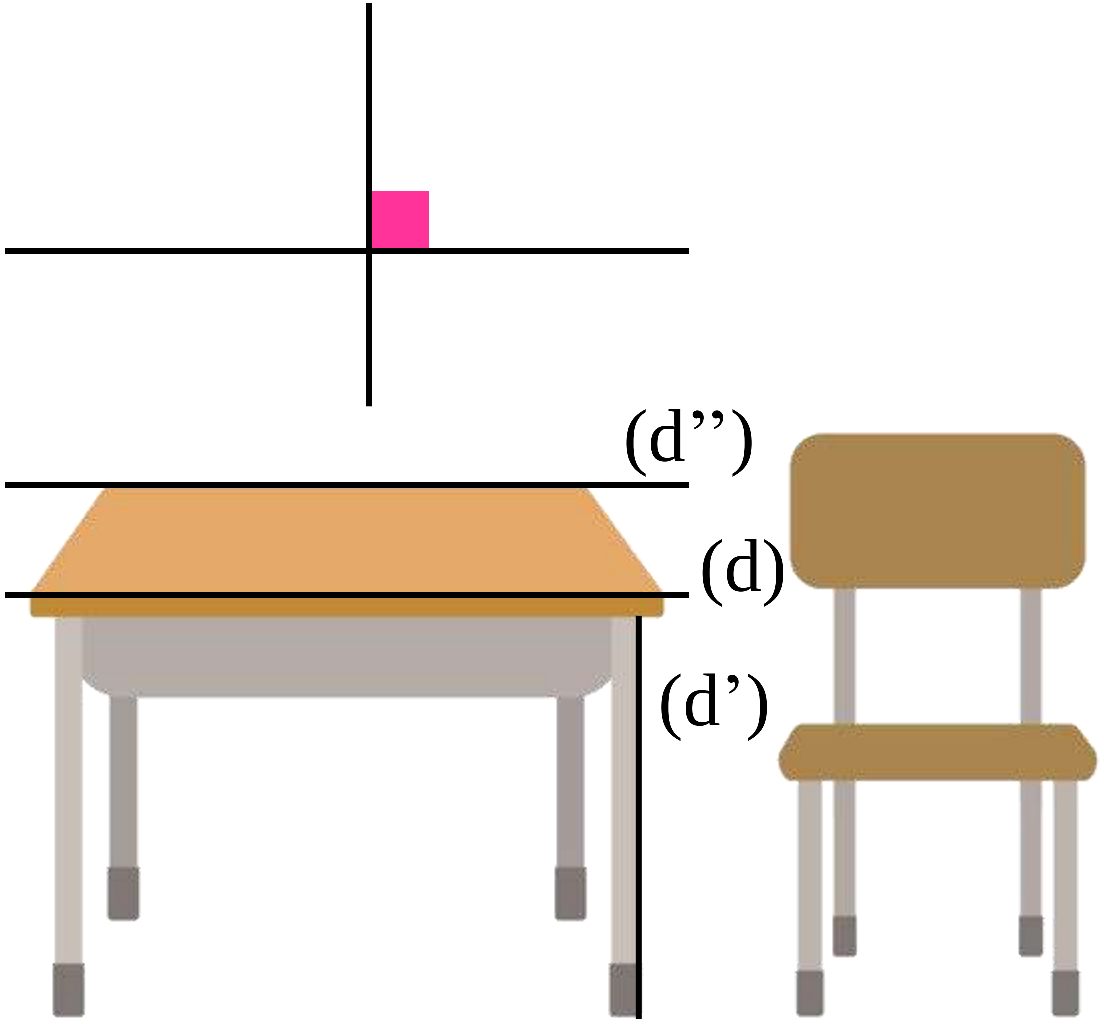
Remark:

If the angle between the two lines is right, so the lines are called orthogonal (⊥).



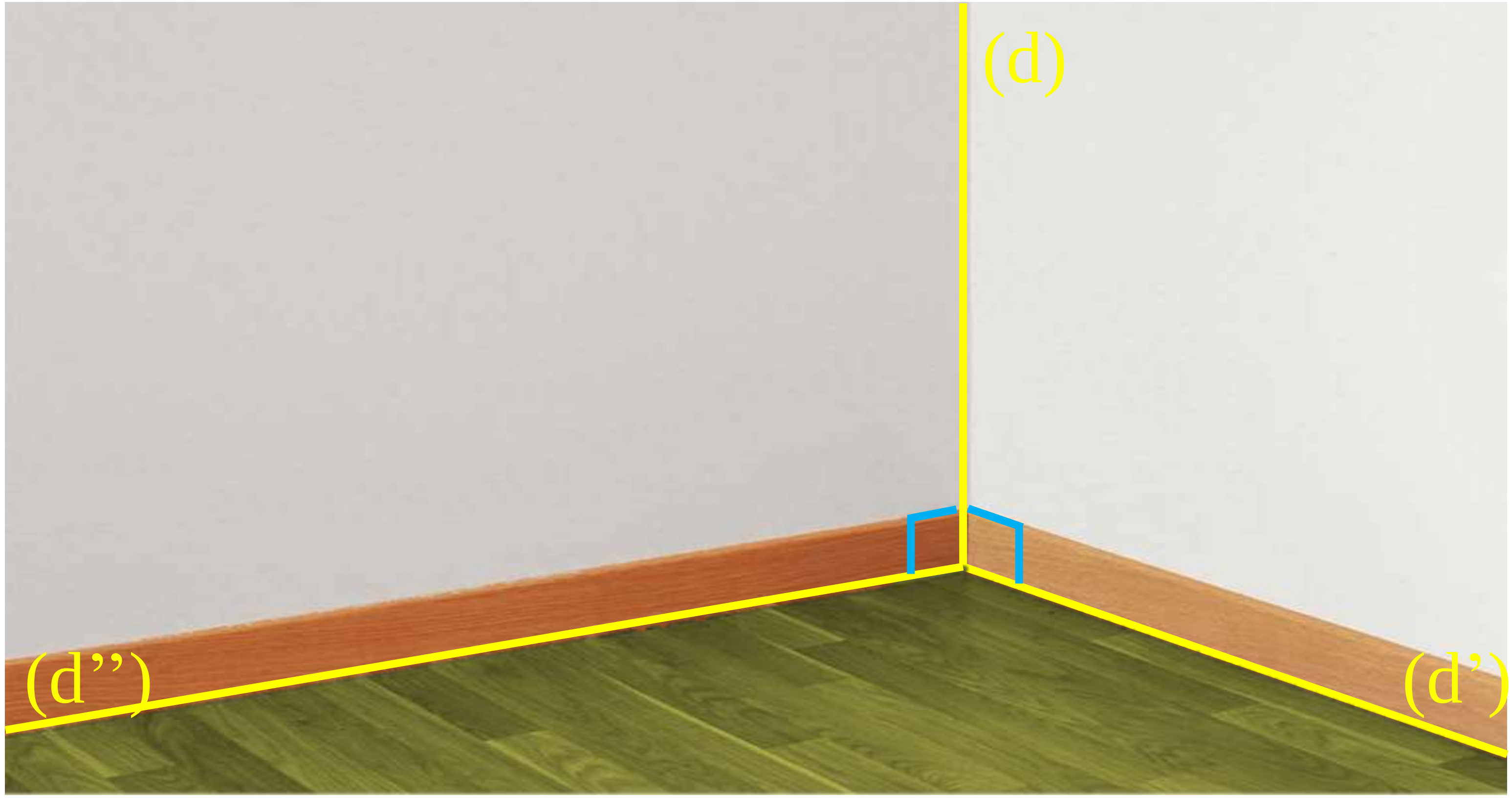
Orthogonality in space
Orthogonality and parallelism

$(d) \perp (d')$
$(d) \parallel (d'')$
} $(d') \perp (d'')$



Orthogonality in space
Line perpendicular to a plane

$$\left.\begin{array}{l}(d) \perp (d') \\ (d) \perp (d'')\end{array}\right\} (d) \perp ((d'),(d''))$$

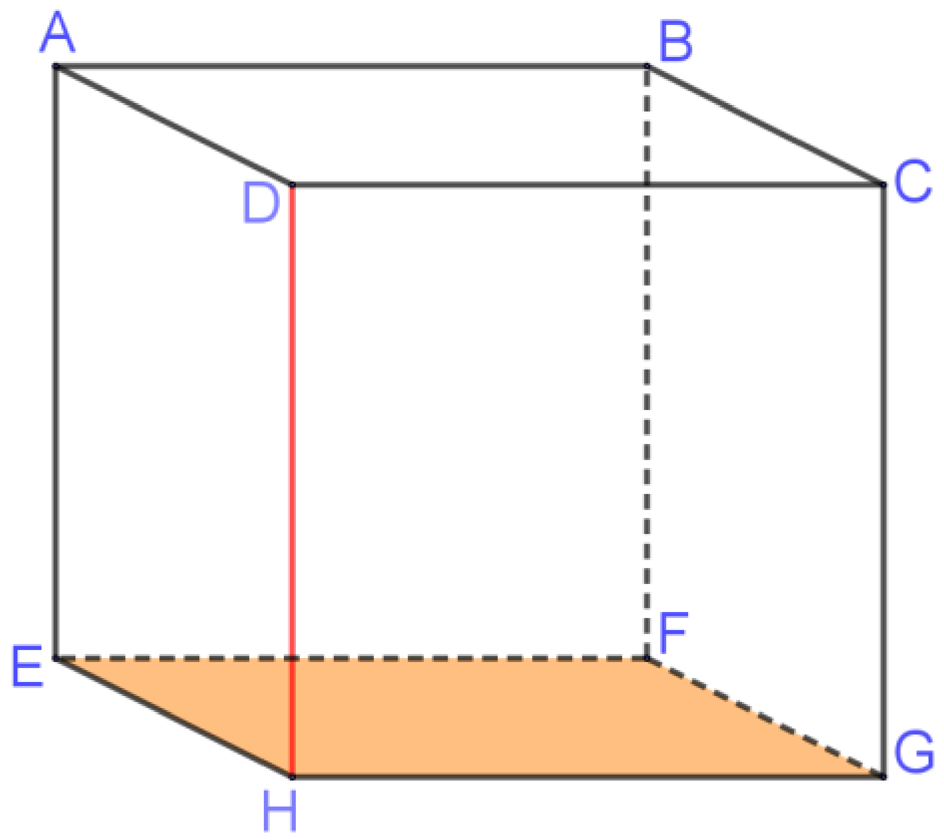


To prove that a line is perpendicular (orthogonal) to a plane (P), it is sufficient to prove that this line is perpendicular (orthogonal) to two intersecting lines of this plane.

Orthogonality in space
Line perpendicular to a plane

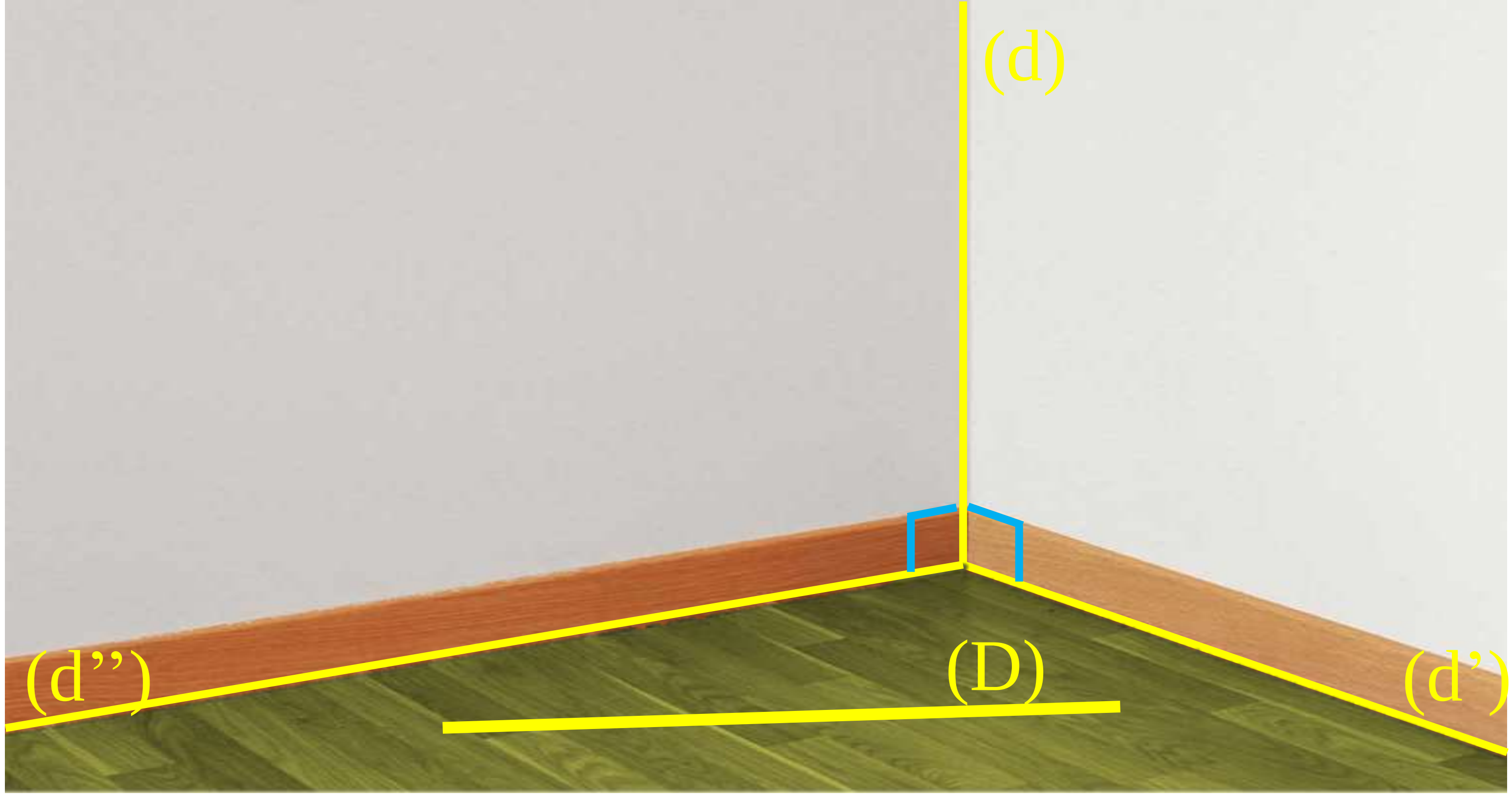
Example:

ABCDEFGH is a cube of edge a.
Show that (DH) is perpendicular to the plane (EFG).



(EFG) = (EFGH)
(DH) ⊥ (HG) (DCGH is a square)
(DH) ⊥ (EH) (ADHE is a square)
So (DH) ⊥ ((HG),(EH)) = (EFG)

Orthogonality in space
Line perpendicular to a plane



$$\left.\begin{array}{l}(d) \perp (P)\\ (D) \subset (P)\end{array}\right\} (d) \perp (D)$$

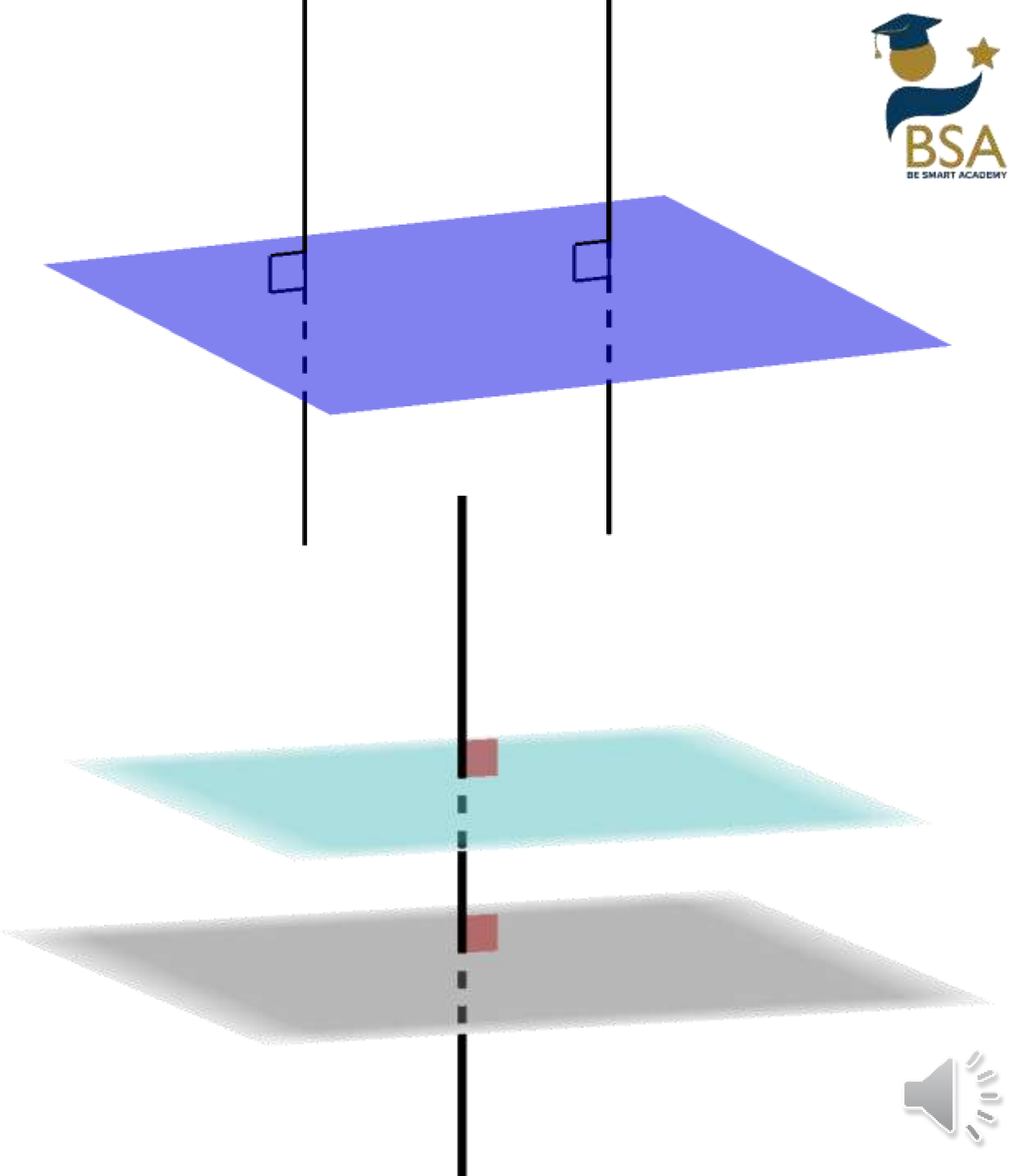
Orthogonality in space
Line perpendicular to a plane

Remark:

❶ If two lines are perpendicular to the same plane, then they are parallel.

❷ If two lines are parallel, then every plane perpendicular to one of them is perpendicular to the second.

❸ if two planes are parallel, any line perpendicular to one of them is perpendicular to the second.



Orthogonality in space

Line perpendicular to a plane

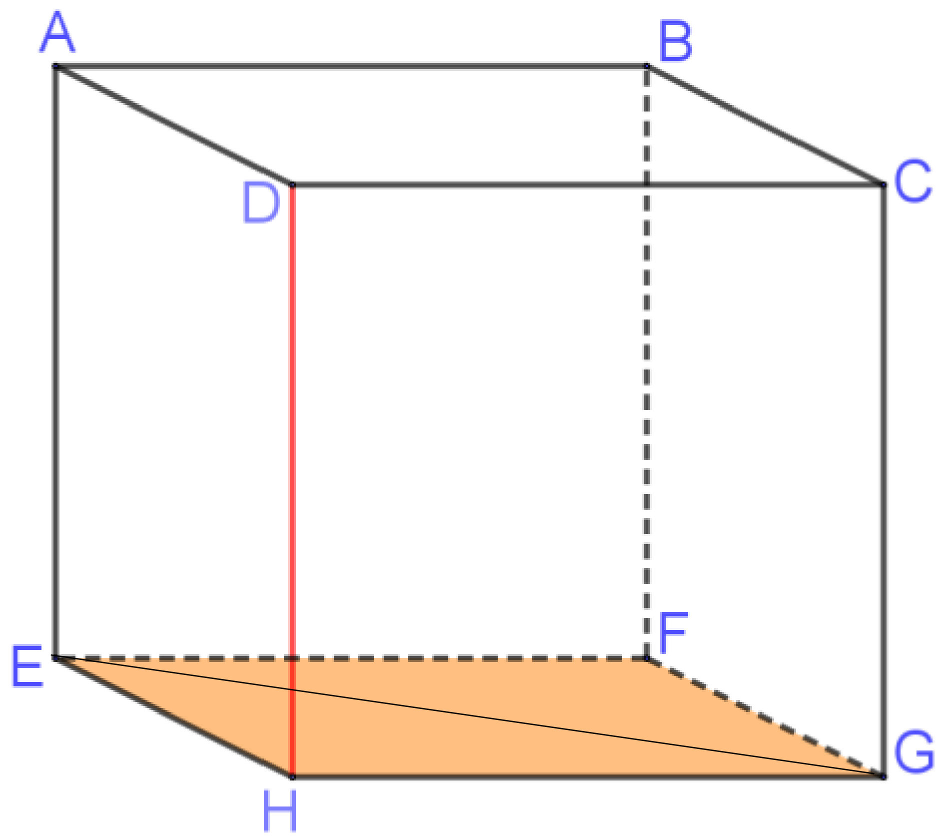
Example:

ABCDEFGH is a cube of edge a.
Show that (DH) is orthogonal to (EG).

We proved before that $(DH) \perp (EFGH)$

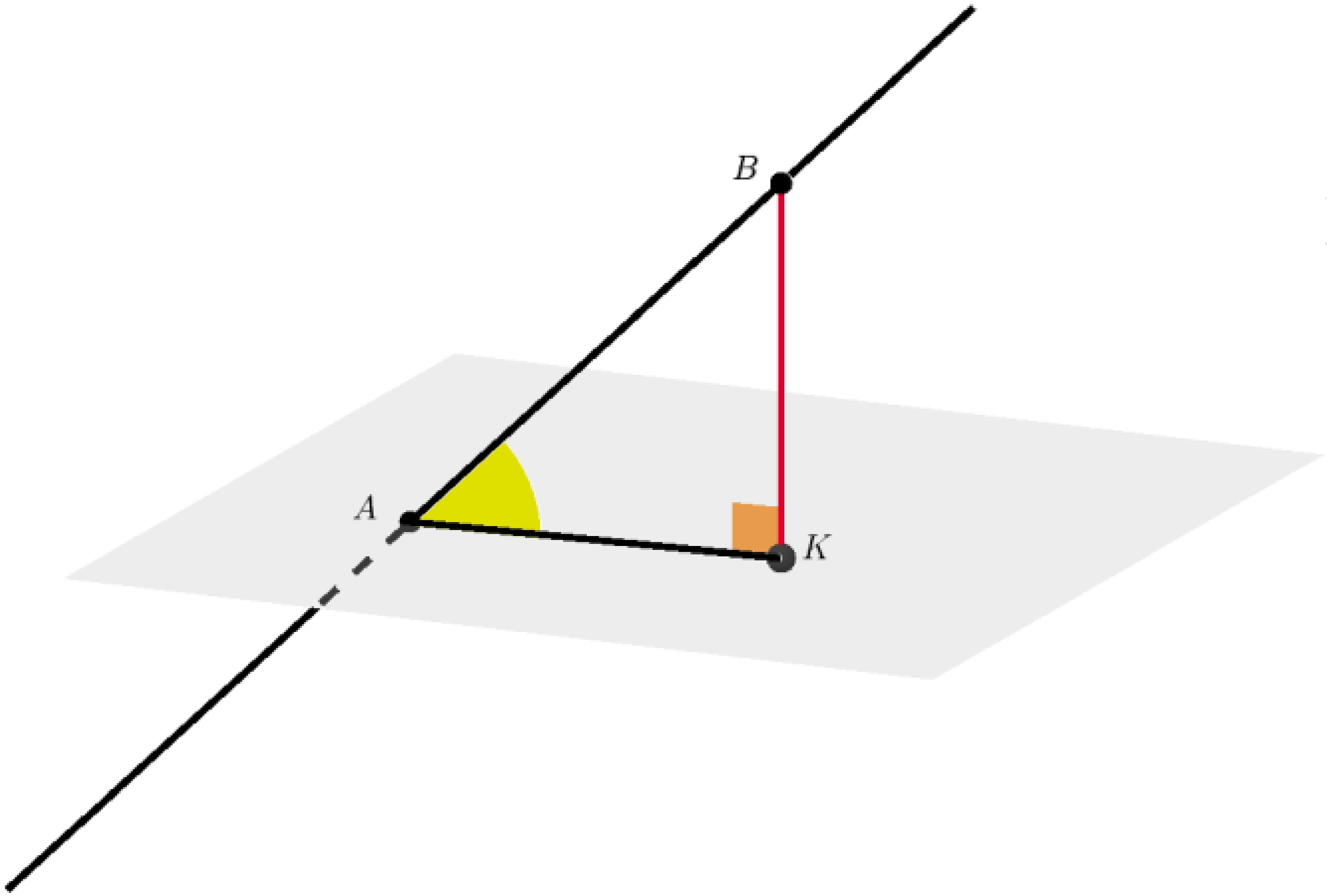
$(EG) \subset (EFGH)$

So $(DH) \perp (EG)$



Orthogonality in space

Angle between line and plane



Step 1: Search a point of the line other than A.

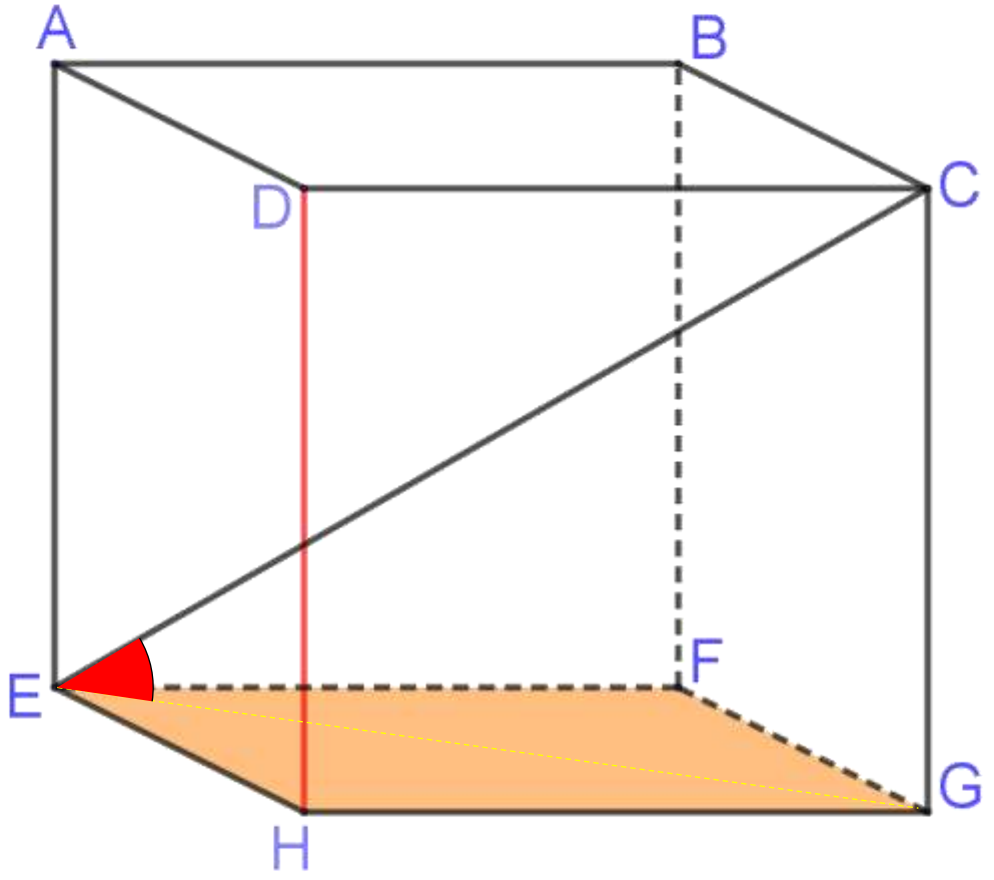
Step 2: Draw the orthogonal projection of B on the plane.

Step 3: Calculate the angle $\widehat{BAK}$.

(d) Cuts (P) at A
(BK) ⊥ (P)
Then $(\widehat{(d),(P)}) = \widehat{BAK}$

Orthogonality in space
Angle between line and plane
Example:

ABCDEFGH is a cube of edge a.
Find the angle between (EC) and the plane (EFGH).



{E}=(EC)∩(EFGH)

(GC) ⊥ (HG)

(GC) ⊥ (FG)

Then (GC) ⊥ (EFGH) at G

So $\left(\widehat{(EC),(EFGH)}\right) = \widehat{CEG}$

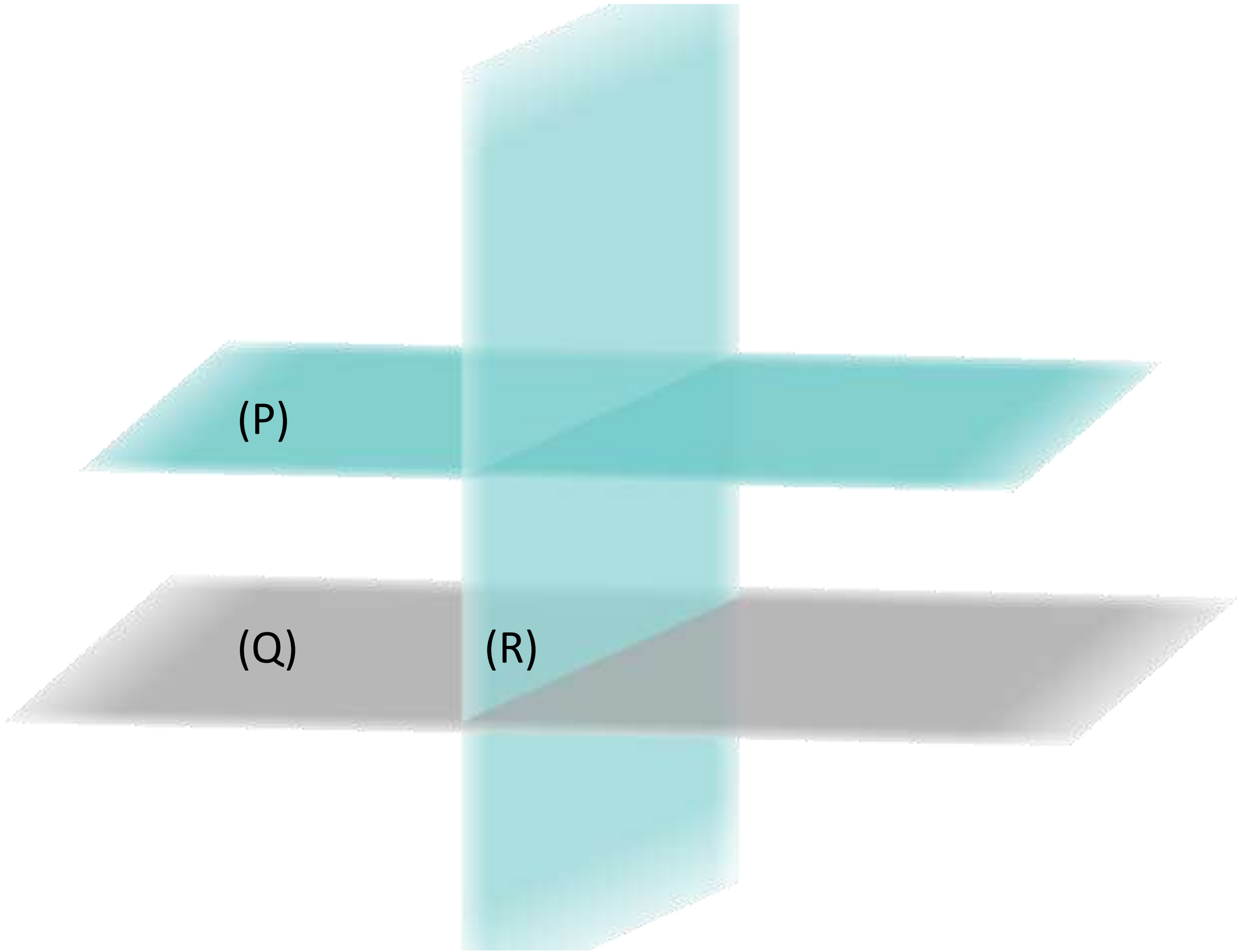
(CG) ⊥ (EG) since ((EG)⊂(EFGH))

In the right triangle CEG at G:

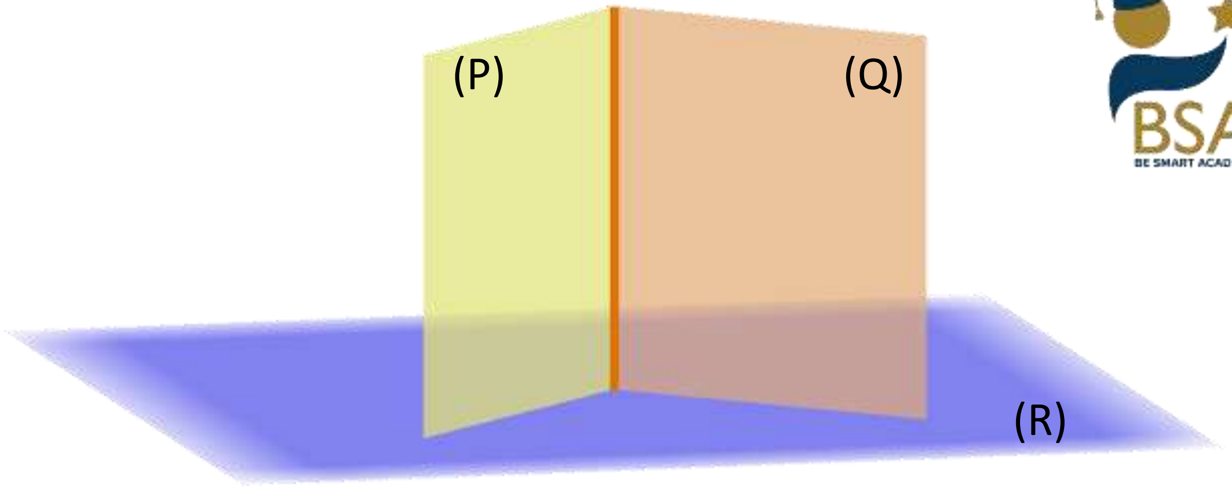
$\tan\widehat{CEG} = \frac{CG}{EG} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$ so $\widehat{CEG} = \tan^{-1}\frac{1}{\sqrt{2}} = 35.26°$

Orthogonality in space
Perpendicular planes

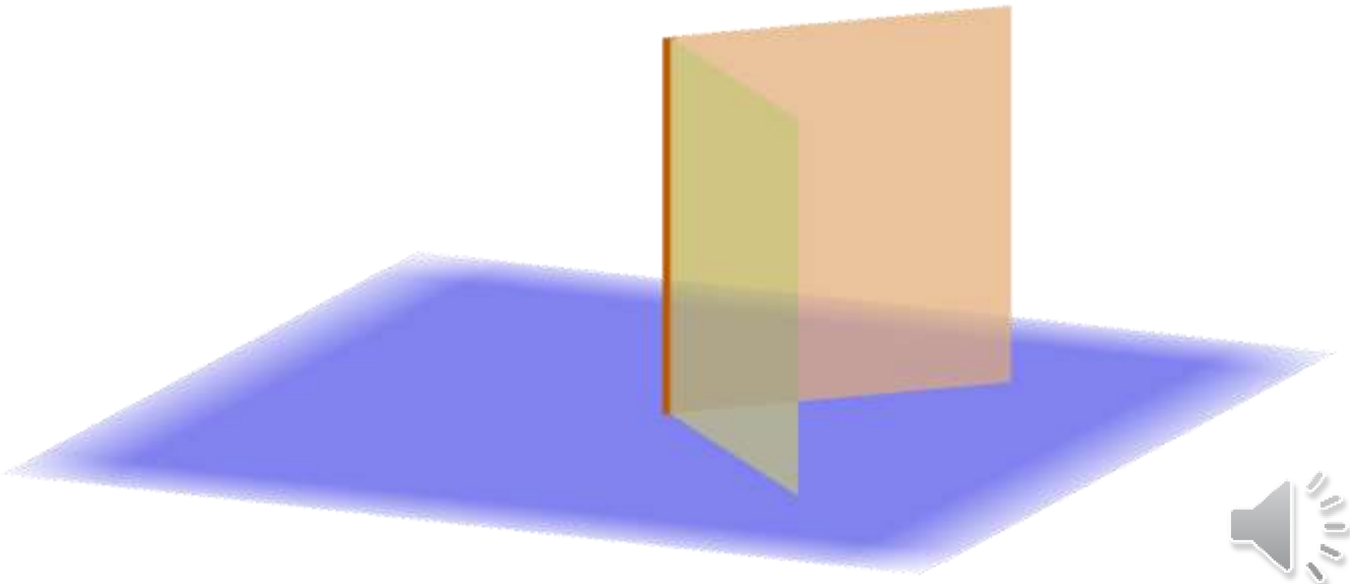
❶ If two planes (P) and (Q) are parallel, any plane (R) perpendicular to one of them is perpendicular to the second.
(P) // (Q)
(P) ⊥ (R)
Then (Q) ⊥ (R)



Orthogonality in space
Perpendicular planes



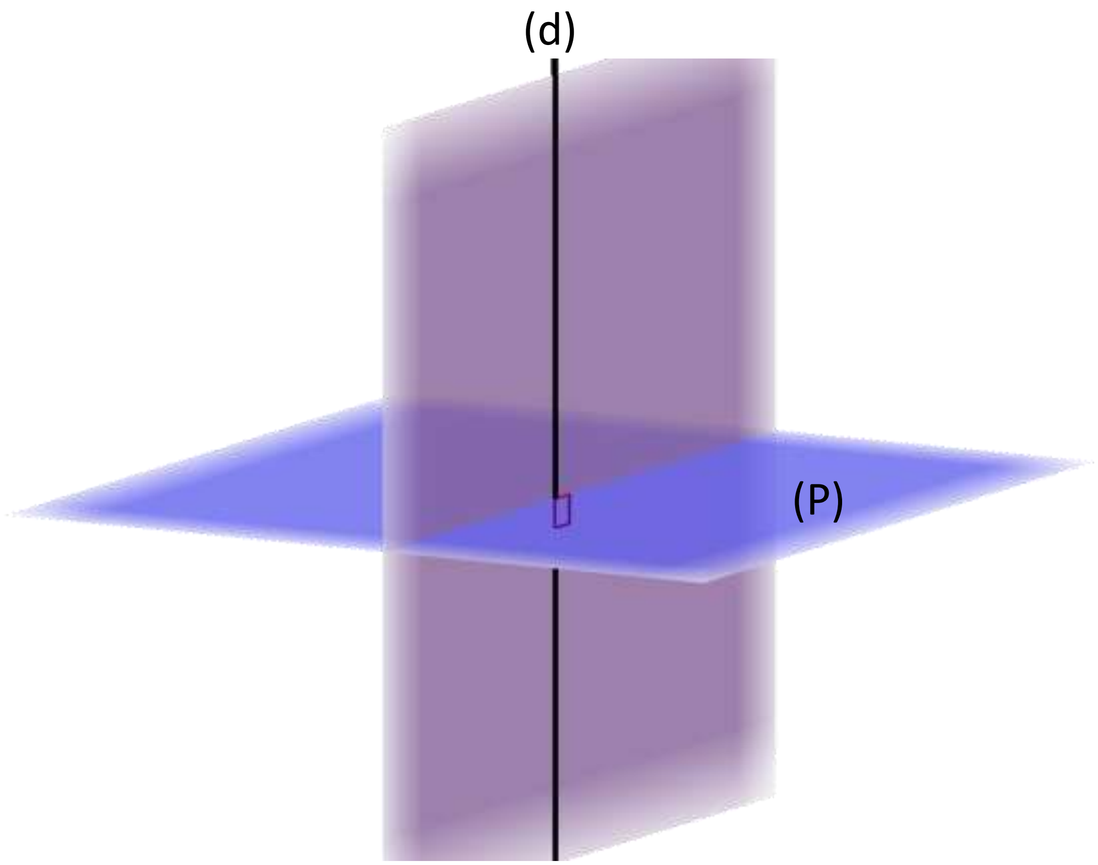
❷ if two planes (P) and (Q) are perpendicular to a plane (R), then their intersection line is perpendicular to (R).

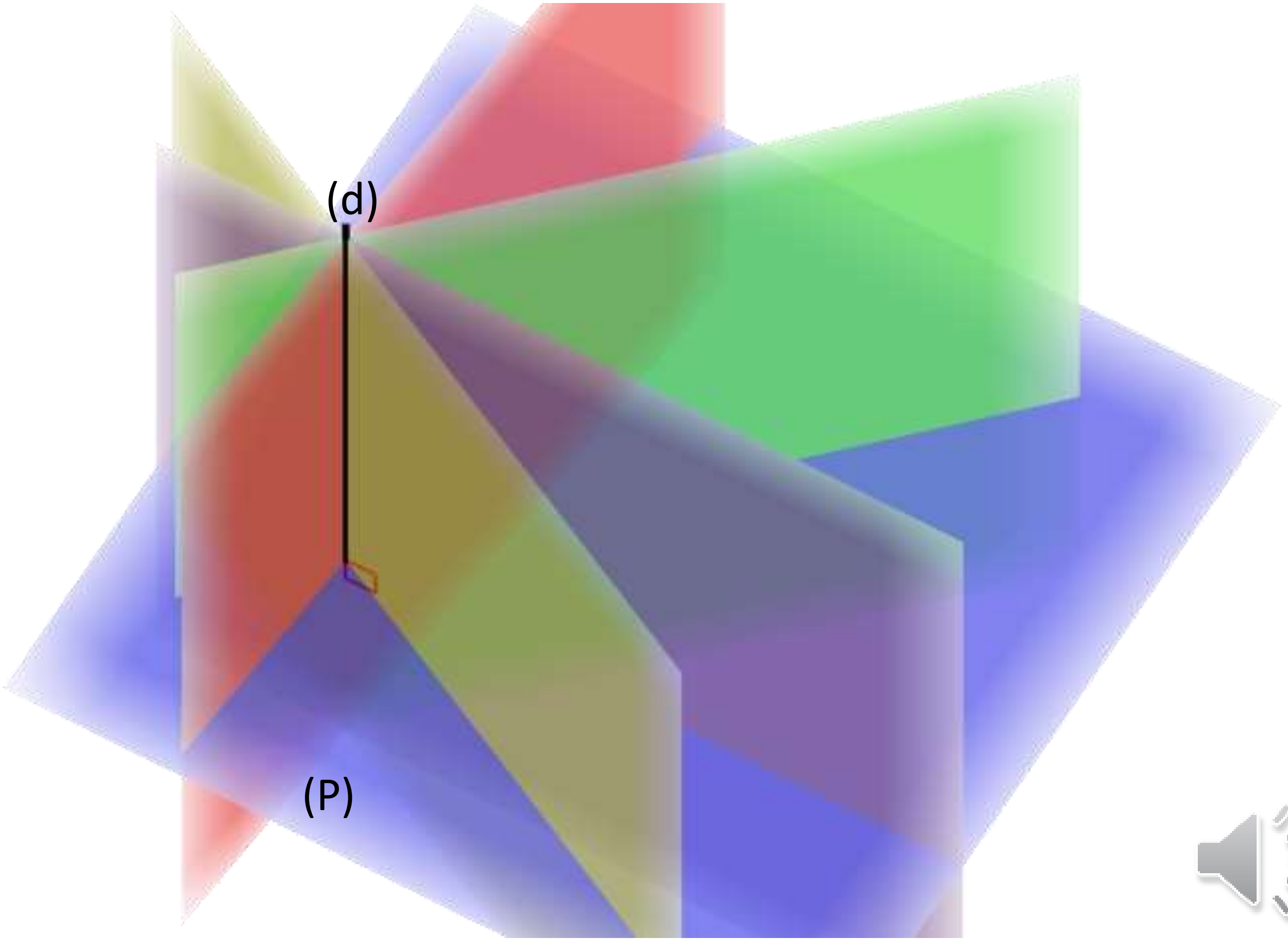


Orthogonality in space

Perpendicular planes

❸ if a line (d) is perpendicular to a plane (P), then any plane contains (d) is perpendicular to (P)





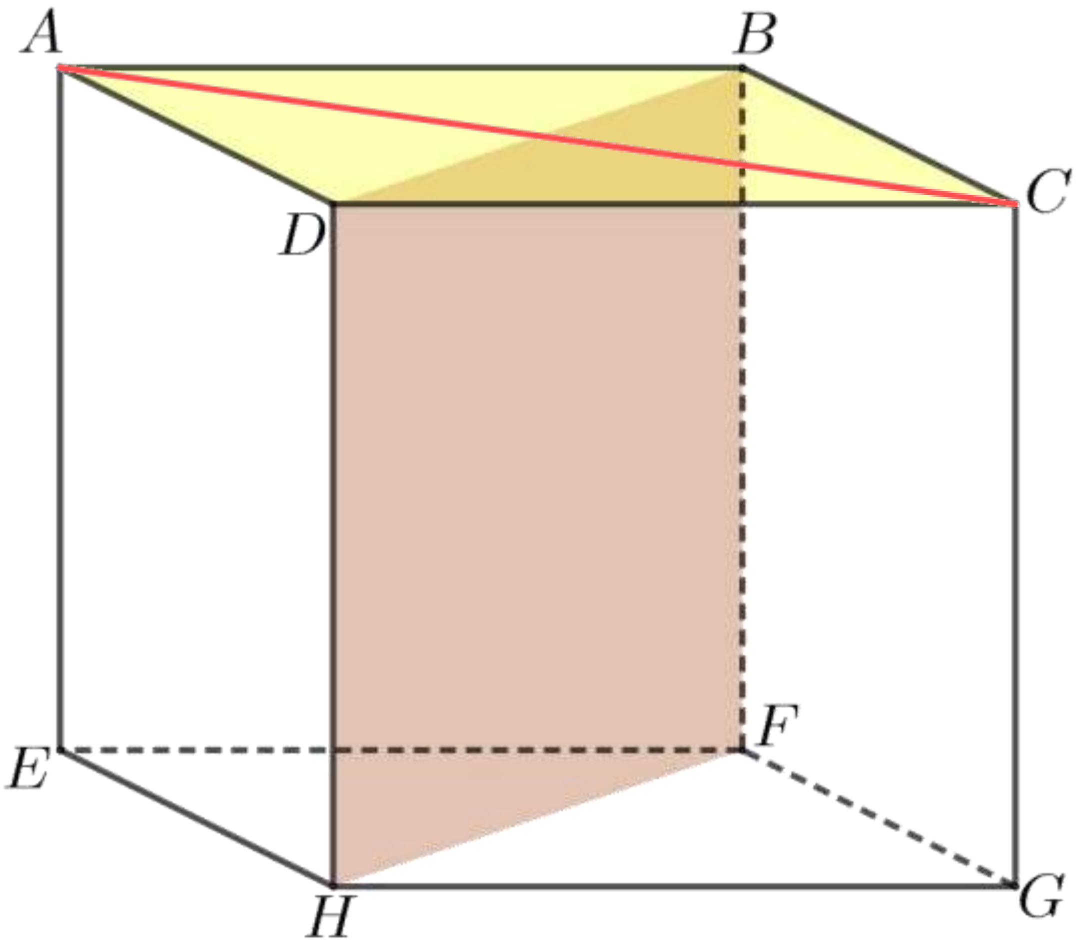
Orthogonality in space

Perpendicular planes

Example:

ABCDEFGH is a cube of edge a.
Show that (ABCD) ⊥ (HDBF).

We proved before that (HD) ⊥ (AC)
(AC) ⊥ (BD) (diagonals in a square are perpendicular).
So (AC) ⊥ (HDBF)
But (AC)⊂(ABCD)
So (ABCD)⊥(HDBF).



Orthogonality in space
Angle between two planes

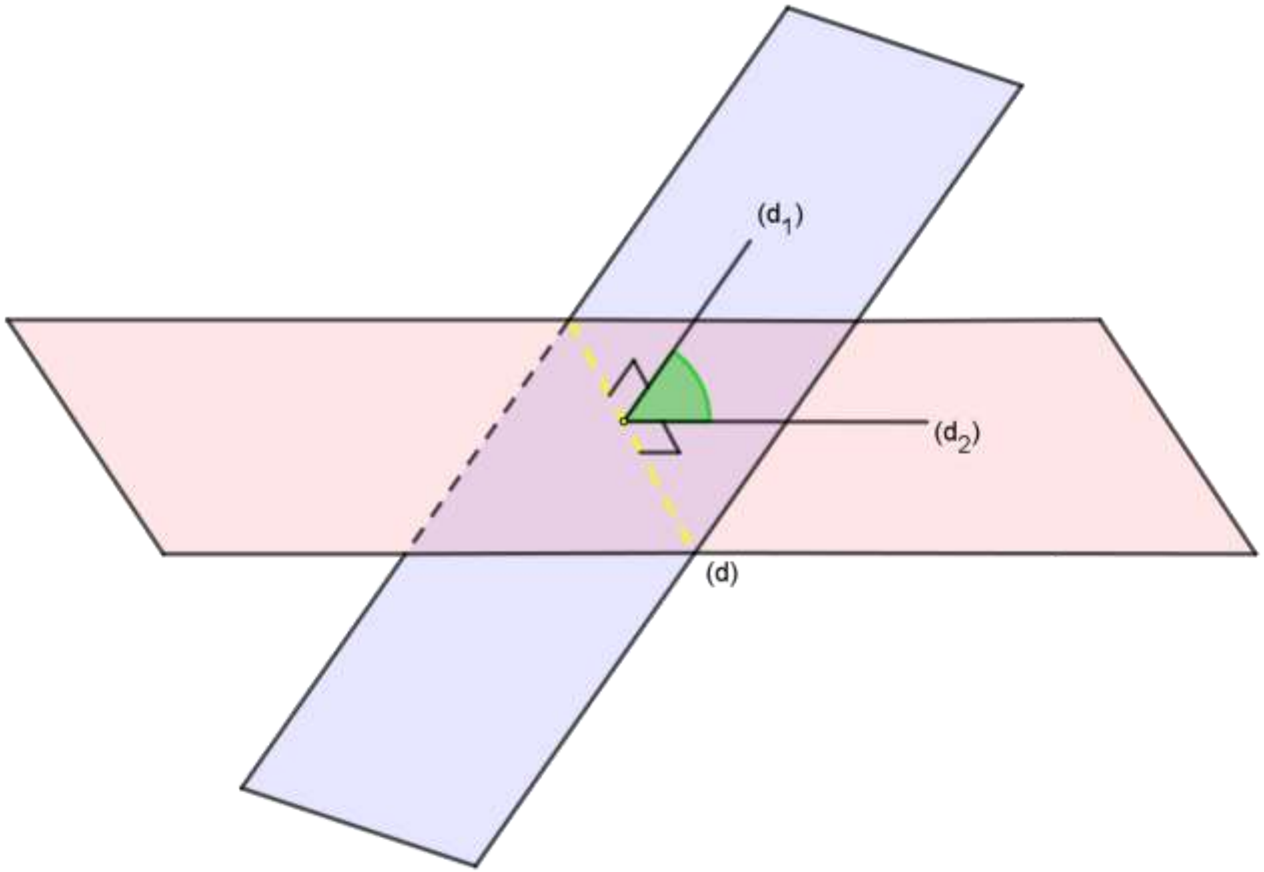
To find the angle between two plane, you must follow the following steps:

❶ Determine the intersection line (d) of the two planes.

❷ Find (d1) a line perpendicular to (d) of the first plane.

❸ Find (d2) a line perpendicular to (d) of the second plane.

❹ the dihedral angle between the two lines (d1) and (d2) is equal to that between the two planes.

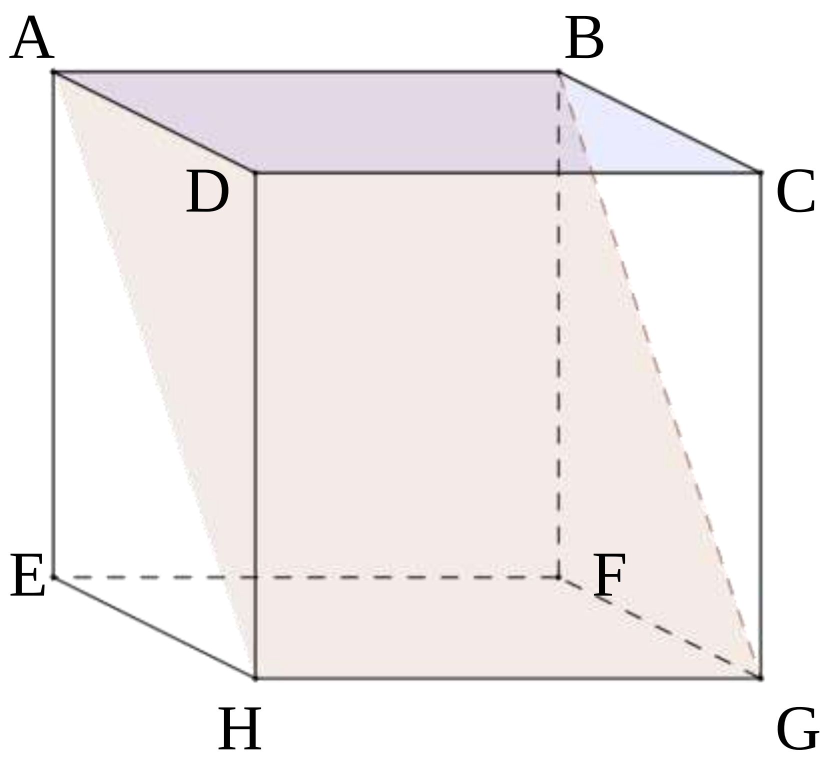


Orthogonality in space

Angle between two planes

Example:

ABCDEFGH is a cube of edge a. Find the angle between the planes (ABCD) and (ABGH).



$(ABCD) \cap (ABGH) = (AB)$

$(AD) \perp (AB)$

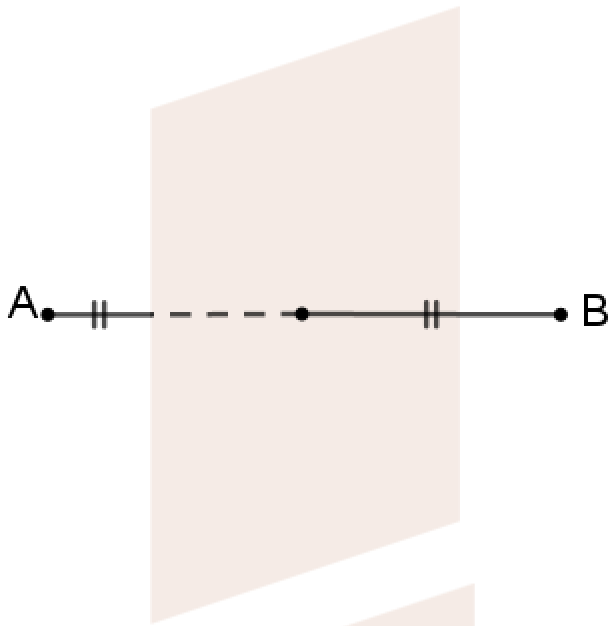
$(AH) \perp (AB)$ since $(AB) \perp$ to the plane (ADHE)

So the dihedral angle between (ABCD) and (ABGH) is the angle between (AD) and (AH) which is $\widehat{HAD}$.

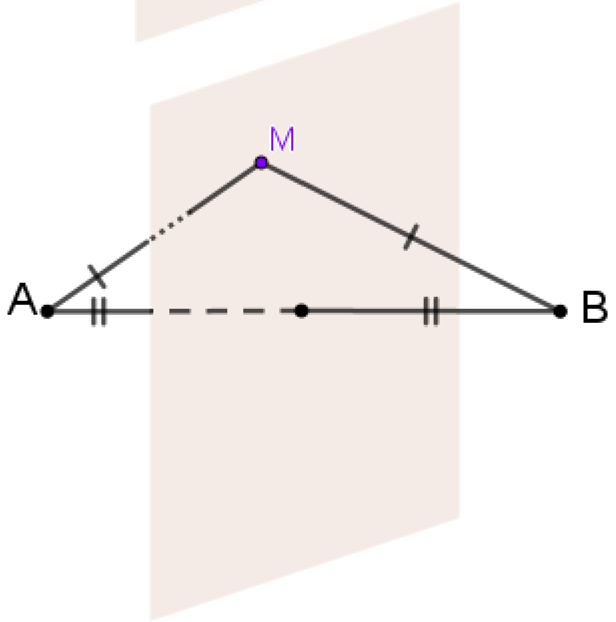
$\widehat{HAD} = 45°$

Orthogonality in space
Mediator plane

(P) Is the mediator plane of [AB]:
- (AB)⊥(P)
- The midpoint of [AB] belongs to (P).



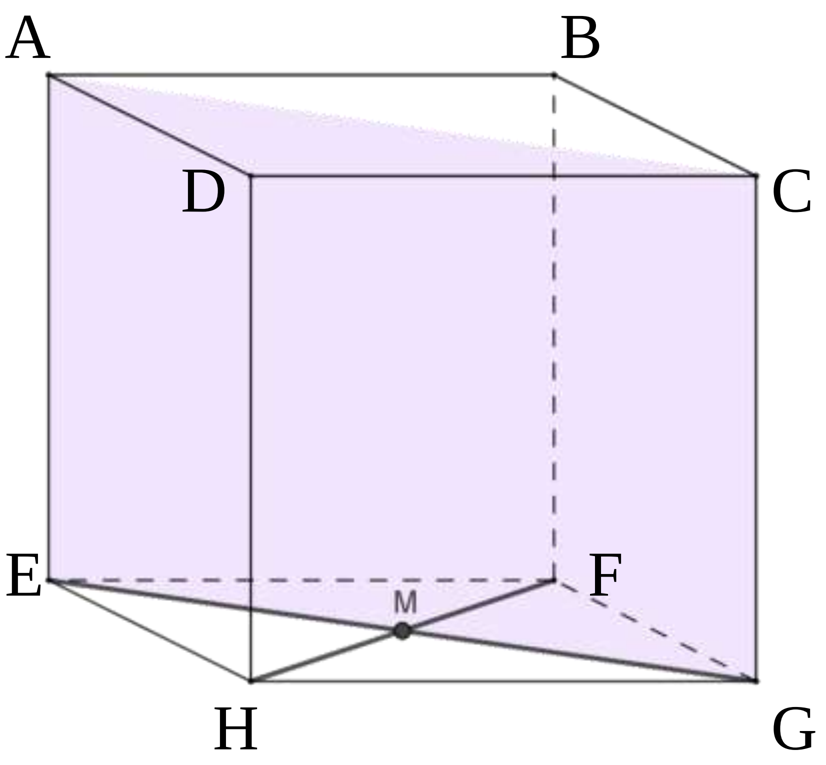
- If M is any point of (P), then MA=MB



Orthogonality in space
Mediator plane

Example:

ABCDEFGH is a cube of edge a.
Show that (ACGE) is the mediator plane of [HF].



AH=AF=$a\sqrt{2}$ so A belongs to the mediator plane of [HF].

EF=EH so E belongs to the mediator plane of [HF].

GF=GH so G belongs to the mediator plane of [HF].

Then the plane formed by A, E and G is the mediator plane of [HF] which is (EFGH).